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REVISION OF PROOF OF THE FORMULA FOR THE ERROR OF OBSERVATION.

BY CHAS. H. KUMMELL, U. S. LAKE SURVEY, DETROIT, MICH.

On page 169, Vol. III, after equation (59′) to the bottom of the page, the following may be substituted:

But we can determine also the probable errors of the resulting corrections, $\Delta_1, \Delta_2, \ldots \Delta_m$, in terms of the probable error of observation.

We have from (47′)

$$\begin{aligned}\Delta_1 = &(a_1\alpha_1 - 1 + b_1\beta_1 + c_1\gamma_1 + \ldots + l_1\lambda_1)\mu_1 \\ &+ (a_1\alpha_2 \quad + b_1\beta_2 + c_1\gamma_2 + \ldots + l_1\lambda_2)\mu_2 \\ &+ \ldots\ldots\ldots\ldots\ldots\ldots \\ &+ (a_1\alpha_m \quad + b_1\beta_m + c_1\gamma_m + \ldots + l_1\lambda_m)\mu_m;\end{aligned}$$

and similarly for the other corrections. Applying theorem (43′) we have, if r_1, r_2, r_m denote the probable errors of the corrections,

$$\begin{aligned}r_1^2 = r^2 \{ &1 - 2(a_1\alpha_1 + b_1\beta_1 + c_1\gamma_1 + \ldots + l_1\lambda_1) \\ &+ \quad a_1^2[\alpha^2] + a_1b_1[\alpha\beta] + a_1c_1[\alpha\gamma] + \ldots + a_1l_1[\alpha\lambda] \\ &+ a_1b_1[\alpha\beta] + \quad b_1^2[\beta^2] + b_1c_1[\beta\gamma] + \ldots + b_1l_1[\beta\lambda] \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &+ a_1l_1[\alpha\lambda] + b_1l_1[\beta\lambda] + c_1l_1[\lambda\gamma] + \ldots + \quad l_1^2[\lambda^2] \} \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (60)\end{aligned}$$

Resolving the first of (58) with respect to $a_1, a_2, \ldots\ldots a_m$, which are independent, we have,

$$\begin{aligned}a_1\alpha_1 &= a_1^2[\alpha^2] + a_1b_1[\alpha\beta] + a_1c_1[\alpha\gamma] + \ldots + a_1l_1[\alpha\lambda], \\ a_2\alpha_2 &= a_2^2[\alpha^2] + a_2b_2[\alpha\beta] + a_2c_2[\alpha\gamma] + \ldots + a_2l_2[\alpha\lambda], \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\end{aligned}$$

Similarly, resolving the second of (58′) with respect to b_1, b_2, b_m,

$$\begin{aligned}b_1\beta_1 &= a_1b_1[\alpha\beta] + b_1^2[\beta^2] + b_1c_1[\beta\gamma] + \ldots + b_1l_1[\beta\lambda] \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\end{aligned}$$

Substituting these expressions into (60) we obtain

$$\begin{aligned}r_1^2 = r^2 \{ &1 - a_1^2[\alpha^2] - a_1b_1[\alpha\beta] - a_1c_1[\alpha\gamma] - \ldots - a_1l_1[\alpha\lambda] \\ &- a_1b_1[\alpha\beta] - \quad b_1^2[\beta^2] - b_1c_1[\beta\gamma] - \ldots - b_1l_1[\beta\lambda] \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &- a_1l_1[\alpha\lambda] - b_1l_1[\beta\lambda] - c_1l_1[\gamma\lambda] - \ldots - \quad l_1^2[\lambda^2] \} \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (60')\end{aligned}$$

Summing these equations we have

$$\begin{aligned}[r^2] = r^2 \{\, m &-[a^2][a^2]-[ab][a\beta]-[ac][a\gamma]-\ldots-[al][a\lambda]\\ &-[ab][a\beta]-[b^2][\beta^2]-[bc][\beta\gamma]-\ldots-[bl][\beta\lambda]\\ &\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\\ &-[al][a\lambda]-[bl][\beta\lambda]-[cl][\gamma\lambda]-\ldots-[l^2][\lambda^2]\,\}\\ = r^2(m&-n), \text{ by virtue of (58), (58}'\text{).} \qquad (60'')\end{aligned}$$

But as already stated the series of corrections, $\Delta_1, \Delta_2, \Delta_3, \ldots\ldots \Delta_m$, conforms to the laws of error as near as the conditions of the problem permit, therefore we may consider it an ideal series. For such a series and not for a series of *true errors* (as most authors do) we can employ formula (53) to obtain its probable error. We have evidently, for an ideal series,

$$[r^2] = mr^2_{\triangle};$$

$$\therefore\ mr^2_{\triangle} = (m-n)r^2. \qquad (60''')$$

We have then the remarkable relation: &c.

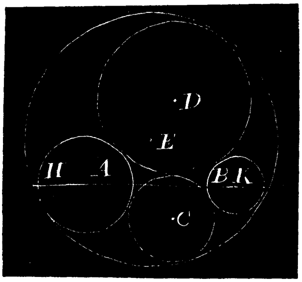
GENERAL SOLUTION OF PROBLEM ON P. 143, VOL. V.

BY E. B. SEITZ, GREENVILLE, OHIO.

Problem.—"It is required to describe upon the same plane, three circles touching each other, each of which shall touch two given circles."

Solution.—The two given circles may be wholly exterior to each other, or one may be wholly within the other. Hence there are two cases.

1. Suppose the construction completed, A, B being the given circles, and C, D, E the required circles. Now since each of the circles A, B touches the three circles C, D, E, which touch each other, the square of the common tangent of A, B is equal to four times the product of their diameters. A demonstration of this theorem has been given by Mr. Turrell in Vol. I, pp. 171–3. The following is a somewhat shorter demonst.



Let a, b, c, d be the radii of the circles A, B, C, D, and t_1, t_2, t_3, t_4, and T, T' the common tangents of A and C, C and B, B and D, D and A, A and B, and C and D.

Then, since A, C, B, D are all touched by the circle E in the same way, according to the principle stated on page 24, Vol. II, we have

$$TT' = t_1t_3 + t_2t_4.$$